

Turbulence and Stress Owing to Gravity Wave and Tidal Breakdown

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It has been suggested (Lindzen, 1967, 1968a, b; Lindzen and Blake, 1971; Hodges, 1969) that turbulence in the upper mesosphere arises from the unstable breakdown of tides and gravity waves. Crudely speaking, it was expected that sufficient turbulence would be generated to prevent the growth of wave amplitude with height (roughly as $(\text{basic pressure})^{-1/2}$). This work has been extended to allow for the generation of turbulence by smaller amplitude waves, the effects of mean winds on the waves, and the effects of the waves on the mean momentum budget. The effects of mean winds, while of relatively small importance for tides, are crucial for internal gravity waves originating in the troposphere. Winds in the troposphere and stratosphere sharply limit the phase speeds of waves capable of reaching the upper mesosphere. In addition, the existence of critical levels in the mesosphere significantly limits the ability of gravity waves to generate turbulence, while the breakdown of gravity waves contributes to the development of critical levels. The results of the present study suggest that at middle latitudes in winter, eddy coefficients may peak at relatively low altitudes (50 km) and at higher altitudes in summer and during sudden warmings (70–80 km), and decrease with height rather sharply above these levels. Rocket observations are used to estimate momentum deposition by gravity waves. Accelerations of about 100 m/s/day are suggested. Such accelerations are entirely capable of producing the warm winter and cold summer mesopauses.

1. INTRODUCTION

The possibility of breaking waves generating turbulence in the mesosphere was noted some years ago by Lindzen [1967; 1968a] and Hodges [1969] for tides and gravity waves, respectively. The idea, in these papers, was simply that vertically propagating gravity waves that, in the absence of damping, grow in amplitude as $p_0^{-1/2}$ (where p_0 is the basic unperturbed pressure) could at some height reach amplitudes for which the wave fields themselves would be strongly unstable (i.e., the combination of mean and wave fields would have a negative static stability or at least its Richardson number would drop below 0.25). Above such a height it was suggested that the waves would generate sufficient turbulence, on the average, to prevent further wave growth with height. Theoretical results implied that among tidal modes, only the first propagating diurnal mode would prove important in this respect. Lindzen and Blake [1971] estimated that this mode would break down between 80 and 90 km, generating turbulence up to a height of about 108 km above which molecular viscosity and conductivity are sufficient to inhibit further wave growth. The turbulence generated by this tidal mode is restricted to within about 30° latitude of the equator as is the mode itself. The cessation of turbulence at 108 km corresponds closely to the height usually designated for the turbopause and in a loose fashion supports the notion of tidal generation for turbulence.

The situation with respect to internal gravity waves is less certain. Structures with moderately short vertical wavelengths (0(12 km)) are commonly observed in rocket soundings, poleward of 30° (thus excluding diurnal tides, at least in some measure), especially in winter. Examples of such soundings are shown in Figure 1. On the basis of dispersion relations given by Hines [1960], one commonly—but arbitrarily—identified the gravity waves as oscillations of relatively short period (0(3 hours)). Observations suggested that such gravity waves also broke down, and Hodges [1969] estimated the resulting generation of turbulence along similar lines to those

discussed in connection with the diurnal tide. Again, the turbulence was assumed to persist up to some turbopause height. These studies tended to ignore the fact that the relevant frequencies were the Doppler shifted frequencies and that intrinsic frequencies might be very different.

Lindzen [1971] presented a crude model of turbulence in the mesosphere that summarized the above mechanisms and, in addition, argued that 'turbulent' diffusion could result from nonbreaking waves (NB tidal modes are simply special cases of internal gravity waves [viz. Lindzen, 1970]). The specific argument was based on wave transiency, but other arguments have been advanced by Weinstock [1976]. This matter is discussed further in a separate note by R. S. Lindzen and J. Forbes (manuscript in preparation, 1981) wherein we consider the cascade of energy from stable waves to waves of sufficiently small vertical wavelengths to permit unstable breakdown. Returning to Lindzen [1971], the model of eddy diffusion presented consisted in a diffusion coefficient that increased as $(p_0)^{-1/2}$ up to a level at which wave breaking occurred, increased markedly at this level (50–90 km), and remained constant up to some turbopause level. Both the magnitude and the distribution of the diffusion coefficient were reasonably compatible with what was called for by measurements of composition [Hunten, 1975; von Zahn et al., 1980] but the range of uncertainty was large. The notion of exponentially increasing (with height) eddy coefficients has, however, gained fairly general acceptance.

Recently, Holton and Wehrbein [1980] have concentrated on the momentum deposition by breaking waves, modeling this by Rayleigh friction (acting to bring zonal flow to zero). To be sure, wave breaking does lead to deposition of wave momentum flux, quite apart from the generation of diffusive turbulence; the two effects are, however, distinct as will be shown in section 2 of this paper. Moreover, in the case of breaking tides, the deposited momentum is not attempting to bring the mean flow to zero [Fels and Lindzen, 1974]. Nevertheless, as was noted by Leovy [1964] and Lindzen [1968b, 1973], if one anticipates that friction is responsible for the reversal of the latitudinal temperature gradient at the meso-

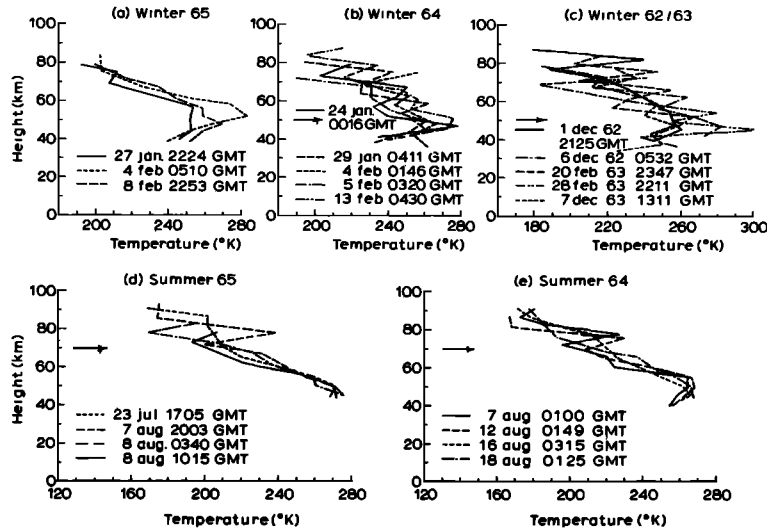


Fig. 1. Winter and summer temperature sounding at Wallops Island (38°N).

pause, then one really does need a mechanism that acts to reduce zonal velocities rather than gradients.

The purpose of the present paper is to reassess carefully, but simply, the role of wavebreaking in the mesosphere, emphasizing the role of the mean flow in determining the behavior of gravity waves. Assuming that most high latitude gravity waves originate in the troposphere, it is shown in section 3 that changes in the mean zonal flow accompanying changing seasons, sudden warmings, etc., dramatically alter the nature of gravity waves in the mesosphere. Such changes lead to simple explanations of various mesospheric observations.

2. GRAVITY WAVES IN THE MESOSPHERE—SIMPLE WKB ANALYSIS

In this section, gravity waves typical of middle to high latitudes will be emphasized, though theoretical results are easily extended to tidal gravity waves. It will be assumed that the reader is familiar with the mathematical theory of internal gravity waves. Only selected results will be presented here. The following references are but a few of the references where more complete treatments are presented: *Hines* [1960], *Lindzen* [1970], *Gossard and Hooke* [1975], and *Holton* [1979].

We will begin by presenting some data relevant to our subsequent discussion. Figure 1 shows various temperature profiles over Wallops Island (38°N) taken in winter and summer by using rocket grenades [*Theon et al.*, 1967]. We will choose to interpret the pronounced waviness in these profiles as being due to internal gravity waves. Winter 1965 results appear anomalous. However, the remaining profiles suggest a fairly clear picture. Vertical wavelengths are typically 0(12 km) in both winter and summer (the coarse resolution of the grenade method leaves such estimates fairly uncertain). Below breaking levels there is uncertainty over wavelength because wave amplitudes are weak.

In winter the profiles begin to manifest neutral or even unstable lapse rates above 50 km; in summer such lapse rates are found above 70 km. Such levels are marked in Figures 1 and 3. We will identify these lapse rates with the onset of wave breaking. In Figure 2 we show the 'typical' distribution of zonal wind with the latitude and height for winter and summer (CIRA, 1972). Of course, it is by now recognized that the results in Figure 2 are not likely to be applicable to particular

winters and summers. For example, in Figure 3 we show winter and summer wind profiles for the latitude band 30°–45°N based on rocket data for the period 1960–1964 [*U.S. Standard Atmosphere Supplement*, 1966]. Also shown are geostrophic winds based on observed temperatures. There is qualitative similarity to Figure 2, but the quantitative differences are obvious. In particular, Figure 3 shows wind magnitudes peaking at the levels of wavebreaking deduced from Figure 1. For our calculations we will use the results in Figure 3. For reference purposes, we show in Figure 4 standard models for the height distribution of temperature in winter and summer (CIRA, 1972). The information in Figure 4 will be needed in subsequent calculations. However, it should be noted that in the absence of wave perturbations, mean lapse rates are only 0 ($\pm 2.4^\circ/\text{km}$) which are (for many purposes) small when compared with $(g/c_p) = 9.8^\circ/\text{km}$.

We now proceed to use the mathematical theory of internal gravity waves to exploit the above observationally derived features. The crudeness of these features leads us to restrict ourselves to only the simplest, approximate, theoretical results. Our equation for the vertical velocity perturbation, w , associated with internal gravity waves is

$$\ddot{w}_{zz} + \left(\frac{N^2(1 + l^2/k^2)}{(\bar{u} - c)^2} - \frac{1}{4H^2} \right) \dot{w} = 0 \quad (1)$$

where

- w vertical velocity perturbation;
- $\dot{w}(z) = \dot{w}(z) e^{z/2H} e^{ik(x-c)} \cos(l y + \phi)$;
- z height;
- H scale height $= RT/g$;
- $\bar{T}(z)$ basic temperature;
- $\bar{u}(z)$ basic zonal wind;
- k wave number in x direction;
- l wave number in y direction;
- x west-east distance;
- y south-north distance;
- c phase speed in x direction;
- g gravitational acceleration;
- N^2 (Brunt-Vaisala frequency) $^2 = g/\bar{T} \Gamma$;
- Γ $(\partial \bar{T} / \partial z) + (g/c_p)$;
- ϕ arbitrary phase.

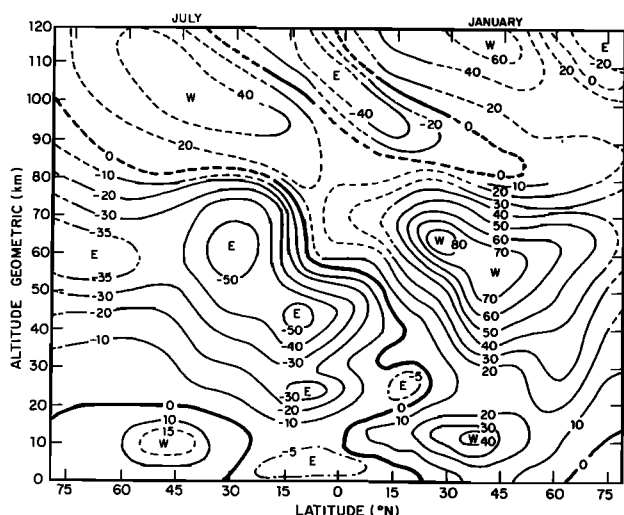


Fig. 2. Zonal wind contours as a function of height and latitude for July and January (CIRA, 1972).

Equation (1) assumes that \bar{u}_{zz} is not too large. Equation (1) has the following approximate WKB solution [viz. Bender and Orszag, 1978]:

$$\bar{w} \approx A\lambda^{-1/2} \exp \{i \int \lambda dz\} \quad (3)$$

where $\lambda^2 = [N^2(1 + P/k^2)/(\bar{u} - c)^2] - (1/4H^2)$, and $A = \text{constant}$.

For the problems we are considering

$$\frac{1}{4H^2} \ll \frac{N^2(1 + P/k^2)}{(\bar{u} - c)^2}$$

and

$$\lambda^2 \approx \frac{N^2(1 + P/k^2)}{(\bar{u} - c)^2} \quad (4)$$

(2) and (3) now determine w . We next wish to obtain the temperature perturbation associated with w . From the thermodynamic energy equation one has

$$ik(c - \bar{U})\delta T \approx w\Gamma \quad (5)$$

or by using (3) and (4)

$$\delta T \approx \frac{\Gamma^{1/2} T^{-1/2} A}{g^{1/2} k (1 + P/k^2)^{1/2}} \lambda^{1/2} e^{i \int \lambda dz} e^{z/2H} e^{ik(x-ct)} \cos(ly + \phi) \quad (6)$$

Equation (6) is appropriate below the level of wave breakdown. For some purposes it proves useful to allow the multiplicative constant in (6) to have a parametric dependence on latitude. It is also convenient to incorporate

$$-ig^{1/2} k \left(1 + \frac{P}{k^2}\right)^{1/2}$$

into the 'constant':

$$\delta T \approx A(\theta)\Gamma^{1/2} T^{-1/2} \lambda^{1/2} e^{i \int \lambda dz} e^{z/2H} e^{ik(x-ct)} \cos(ly + \phi) \quad (7)$$

where $\theta = \text{latitude}$.

For the waves shown in Figure 1, the factor $e^{i \int \lambda dz}$ dominates

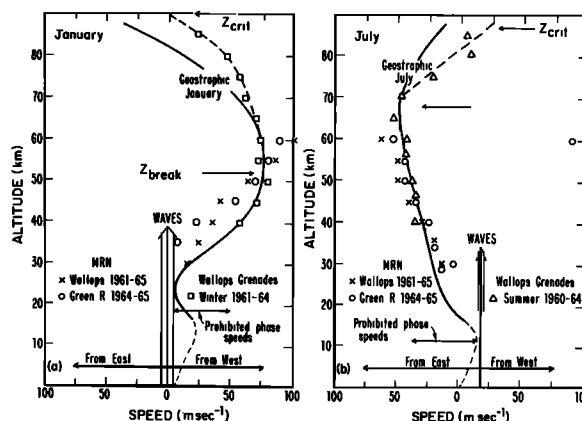


Fig. 3. Zonal wind as a function of height in the latitude belt 30°-45°N based on rocket probes. (a) Results for January; (b) results for July. Also shown are the geostrophic winds derived from temperature soundings and the phase speeds permitted and prohibited for mesospheric gravity waves of tropospheric origin.

the z variation of δT , so

$$\frac{d\delta T}{dz} \approx iA(\theta)\Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{i \int \lambda dz} e^{z/2H} e^{ik(x-ct)} \cos(ly + \phi) \quad (8)$$

For simplicity, we will identify the level of wave breaking with that level at which

$$\left| \frac{d\delta T}{dz} \right| = \Gamma \quad (9)$$

or from (8)

$$A(\theta)\Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{z_{\text{break}}/2H} \approx \Gamma \quad (10)$$

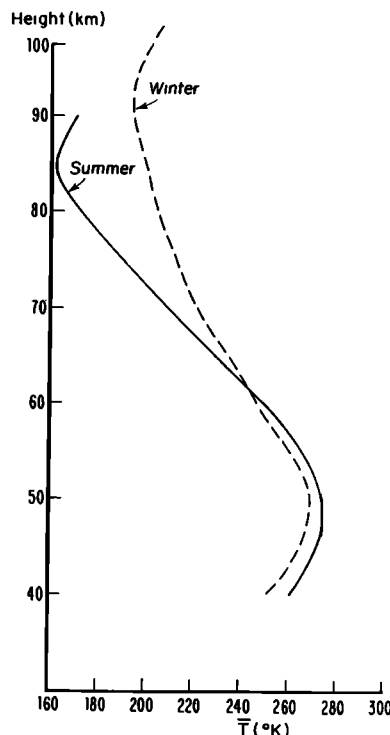


Fig. 4. Zonally averaged temperature as a function of height at 40°N for winter and summer (CIRA, 1972).

Given $A(\theta)$, (10) determines z_{break} , or, alternately, we may use z_{break} as found in Figure 1 to determine $A(\theta)$.

We now hypothesize as in Lindzen [1968] and Hodges [1969] that above z_{break} , sufficient diffusive turbulence is generated to prevent further growth of $|d\delta T/dz|$. Because of this turbulence, the momentum flux carried by the wave will also be deposited in the mean flow above z_{break} . Our determination of the turbulent eddy coefficient follows the treatment of Hodges [1969]. The most important factors in (8) governing the growth of $|d\delta T/dz|$ are $\lambda^{3/2}$ and $e^{z/2H}$, i.e.,

$$\left| \frac{d\delta T}{dz} \right| \propto \lambda^{3/2} e^{z/2H} \tag{11}$$

The $\lambda^{3/2}$ factor causes growth as $|u - c| \rightarrow 0$, and diminution as $|u - c|$ increases. This factor, as a rule, is competitive with the $e^{z/2H}$ growth only as $|u - c| \rightarrow 0$. Here, $\lambda^{3/2}$ can be viewed as adding to the $1/2H$ exponential growth rate an amount given by

$$\frac{1}{\lambda^{3/2}} \frac{d\lambda^{3/2}}{dz} = -\frac{3}{2} \frac{1}{(\bar{u} - c)} \frac{d\bar{u}}{dz} \tag{12}$$

Thus, one wants to determine the value of eddy diffusion that will cancel exponential growth with a rate

$$\frac{1}{2H} - \frac{3}{2} \frac{1}{(\bar{u} - c)} \frac{d\bar{u}}{dz} \tag{13}$$

The effect of eddy diffusion is approximated, following Pitteaway and Hines [1963], by linear damping. Thus in the equations for temperature and horizontal momentum, diffusion contributes terms of the form

$$D_{\text{eddy}} \frac{\partial^2}{\partial z^2} \left\{ \begin{matrix} u \\ \delta T \end{matrix} \right\} \approx -\lambda^2 D_{\text{eddy}} \left\{ \begin{matrix} u \\ \delta T \end{matrix} \right\} \tag{14}$$

The presence of the right-hand side of (14) is equivalent to including an imaginary contribution to c given by

$$kc_i \approx \lambda^2 D_{\text{eddy}} \tag{15}$$

Such a contribution leads to an imaginary contribution to the vertical wave number λ :

$$\lambda \approx \frac{N(1 + P/k^2)^{1/2}}{\bar{u} - c} \approx \frac{Nk(1 + P/k^2)^{1/2}}{k(\bar{u} - c) + i\lambda_r^2 D_{\text{eddy}}}$$

where

$$\lambda_r \approx \frac{N(1 + P/k^2)^{1/2}}{(\bar{u} - c)}$$

Solving for λ , we get the following approximate expression

$$\lambda \approx \frac{D_{\text{eddy}} \lambda_r^3}{k(\bar{u} - c)} \approx \frac{D_{\text{eddy}} N^3 (1 + P/k^2)^{1/2}}{k(\bar{u} - c)^4} \tag{16}$$

The condition that eddy diffusion prevent the growth of $|d\delta T/dz|$ implied by (13) is simply

$$|\lambda_i| \approx \frac{D_{\text{eddy}} N^3 (1 + P/k^2)}{k|\bar{u} - c|^4} \approx \frac{1}{2H} - \frac{3}{2} \left(\frac{1}{\bar{u} - c} \frac{d\bar{u}}{dz} \right)$$

or

$$D_{\text{eddy}} \approx \frac{k|\bar{u} - c|^4}{N^3(1 + P/k^2)^{3/2}} \left[\frac{1}{2H} - \frac{3}{2} \left(\frac{1}{\bar{u} - c} \frac{d\bar{u}}{dz} \right) \right] \tag{17}$$

Here, (17) is appropriate above $z = z_{\text{break}}$. Below z_{break} we still anticipate that gravity waves can generate turbulence [viz. Lindzen, 1971; Weinstock, 1976], but at a lower rate. While we will discuss (17) in more detail in the following section, two points should be mentioned immediately:

1. If $|\bar{u} - c| \gg |\bar{u}|$ (as is the case for the diurnal tide) then we expect D_{eddy} to be almost constant above z_{break} (until molecular viscosity becomes sufficient to damp the wave, a level Lindzen and Blake [1971] associate with the turbopause).

2. If $|\bar{u} - c| \rightarrow 0$ above z_{break} , then we expect D_{eddy} to decrease rapidly as the critical level (where $|u - c| = 0$) is approached.

The last item in our theoretical development is the determination of the deposition of wave momentum flux. In the context of the present discussion, this is given simply by $-d/dz(\rho_0 \bar{u} \bar{w})$, where u and w are the wave zonal and vertical velocity components. We already have an expression for w . Moreover, u is simply related to w . For the vertical wavelengths in Figure 1 we can simplify the continuity equation to

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \approx 0 \approx \text{horizontal divergence} + i\lambda w \tag{18}$$

Now horizontal divergence $\approx ik(1 + P/k^2)u$ [Lindzen, 1970]. Thus

$$u \approx \frac{\lambda}{k(1 + P/k^2)} w$$

and

$$\bar{w}\bar{u} \approx \frac{1}{2} \frac{\lambda}{k(1 + P/k^2)} |w|^2 \tag{19}$$

From (5) and (7) we have

$$|w| \approx g^{1/2} k(1 + P/k^2)^{1/2} A(\theta) \lambda^{1/2} e^{z/2H} \tag{20}$$

for $z < z_{\text{break}}$, and

$$\bar{w}\bar{u} \approx \frac{1}{2} gkA^2(\theta) \frac{\lambda}{|\lambda|} e^{z/H} \tag{21}$$

for $z < z_{\text{break}}$.

Equation (21) is consistent with the Eliassen-Palm Theorem, which states that $d/dz(\rho_0 \bar{u} \bar{w}) = 0$ in the absence of forcing, dissipation, and critical levels. As noted in connection with (10), δT and w are known at z_{break} . It is readily shown that

$$\bar{w}\bar{u}|_{z=z_{\text{break}}} \approx \frac{k}{2} \frac{N^2}{\lambda^3} \tag{22}$$

Above z_{break} , diffusion essentially eliminates the exponential growth shown in (21). Thus, above z_{break}

$$\bar{w}\bar{u} \approx \frac{k}{2} \frac{N^2}{\lambda^3} \Big|_{\text{evaluated at } z_{\text{break}}} \tag{23}$$

Acceleration of the mean zonal flow due to wave breakdown is given by

$$-\frac{1}{\rho_0} \frac{d}{dz} \rho_0 \bar{u} \bar{w} \approx \frac{k}{2H} \frac{N^2}{\lambda^3} \Big|_{z_{\text{break}}} \approx \frac{-k}{2H} \frac{(\bar{u} - c)^3}{N(1 + P/k^2)^{3/2}} \Big|_{\text{eval at } z = z_{\text{break}}} \tag{24}$$

The acceleration will be roughly constant between z_{break} and z_{crit} or some other height beyond which wave action ceases.

While the sign of the acceleration will always be such as to bring \bar{u} toward c , it is clear that the process is not properly described by Rayleigh friction. It is also clear that the mean flow acceleration does not result from approximating the effect of D_{eddy} on the mean flow. The two effects are related but separate manifestations of wavebreaking. Note that the distribution of wave acceleration cannot be fixed in space but must be determined on the basis of the particular positions of z_{break} and z_{crit} at any given time.

3. QUANTITATIVE CONSIDERATIONS

To evaluate the formulae in section 2, information is needed concerning the phase speeds and wave numbers of the relevant waves. Such quantities are, indeed, well known for tidal modes [Chapman and Lindzen, 1970]. This section, therefore, will concentrate primarily on estimating these quantities for gravity waves.

a. Phase Speeds

The determination of phase speeds depends, fairly obviously, on the source of gravity waves. (For tides, thermal forcing is reasonably well known [Lindzen, 1979; Forbes and Garrett, 1979], and the phase speed is essentially the linear rotation speed of the earth, ~ 465 m/s at the equator.) While the possibility of stratospheric and mesospheric origins for mesospheric gravity waves (excluding tides for the moment) exists, no explicit evidence has been found. On the other hand, we do know of very important tropospheric sources, some of which are (1) flow over fixed surface features (mountains, cities, or 'heat islands,' etc.); viz. Queney [1948] and many others); (2) frontal processes [Hines, 1968; Peltier and Ley, 1978]; (3) Kelvin Helmholtz instability [Lindzen and Rosenthal, 1976].

The last two are associated with tropospheric flow speeds, while the first is associated with zero phase speed. However, owing to the unsteadiness of the troposphere, one expects phase speeds actually to be spread somewhat about the above speeds. An explicit study of the gravity wave spectra generated goes well beyond the scope of the present study.

We will simply assume a tropospheric origin for mesospheric gravity waves, and, in light of the above remarks, we will assume the phase speeds to be limited roughly to the range of tropospheric wind speeds (i.e., from some neighborhood of zero to some neighborhood of the tropospheric flow speeds shown in Figure 2).

As concerns the mesosphere, we have an additional (and crucial) constraint. Namely, gravity waves cannot readily pass through critical surfaces (where the flow speed and phase speed are equal). Assuming the zonal flows shown in Figure 3 to be appropriate, at least for discussion purposes (and recognizing that this is not likely to be accurately true at any given time), we see that in winter only phase speeds less than 10 m/s can be transmitted, while in summer only phase speeds somewhat in excess of 20 m/s can be transmitted, at least in the neighborhood of 40° . From Figure 2 we see that lower phase speeds would be permitted at other latitudes. Recall that we do not expect the generation of waves with either large negative phase speeds or with phase speeds much in excess of 20 m/s. Intuitively, we expect much greater generation of gravity wave energy with phase speeds near zero than with phase speeds greater than 20 m/s. This is consistent with the fact

that z_{break} occurs at a much greater height in summer than in winter (viz. Figure 1). On the other hand, during sudden warmings, when easterlies develop in the stratosphere, we expect a significant reduction in the transmission of gravity waves to the mesosphere and a consequent increase in z_{break} . The seasonal variations in z_{break} caused by the seasonal variation in gravity wave filtering by mean winds provides a reasonable explanation of the observed seasonal variation of height of radio reflections from 50 km in winter to 70 km in summer [Balsley *et al.*, 1980]. The above results also appear consistent with the observation of enhanced D region ionization during sudden warmings [Koshelev, 1976].

b. Horizontal Wave Numbers

It is clear from (17) and (24) that our estimates of eddy diffusion and wave-induced acceleration depend on our choice of horizontal wave numbers. Vertical wave numbers are independent of specific choices of horizontal wave numbers, but they do depend on the ratio P/k^2 .

In the case of the main diurnal propagating mode, wave numbers are, of course, well known. From the equivalent gravity mode formulation [Lindzen, 1970]

$$k = 1/6400 \text{ km}$$

and

$$l \approx 6.8k$$

Taking

$$u - c \approx -c \approx 466 \text{ m/s}$$

$$\frac{\partial T}{\partial z} + \frac{g}{c_p} \approx 7.5^\circ/\text{km}$$

and

$$T \approx 250^\circ\text{K}$$

we get from (4)

$$\lambda \approx \left(\frac{g}{T} \left(\frac{\partial T}{\partial z} + \frac{g}{c_p} \right) \right)^{1/2} \left(1 + \frac{P}{k^2} \right)^{1/2} \approx 2.5 \times 10^{-4} \text{ m}^{-1} \quad (25)$$

corresponding to a vertical wavelength of about 25 km as expected.

The situation for gravity waves at middle and high latitudes is more ambiguous. What follows are to some extent 'guessimates.' We will attempt to find a reasonable value of $(l/k)^2$ by matching the 'observed' vertical wave lengths with those predicted by (4). For both winter and summer

$$|u - c| \sim 50 \text{ m/s}$$

$$\frac{\partial T}{\partial z} + \frac{g}{c_p} \approx 7.5^\circ/\text{km}$$

and

$$T \sim 250^\circ\text{K}$$

Now we want

$$\lambda \sim \frac{2\pi}{12 \times 10^3 \text{ m}} \sim 5.236 \times 10^{-4} \text{ m}^{-1}$$

Equation (4) yields

$$\lambda \sim 3.4 \times 10^{-4} \text{ m}^{-1} \left(1 + \frac{P}{k^2} \right)^{1/2}$$

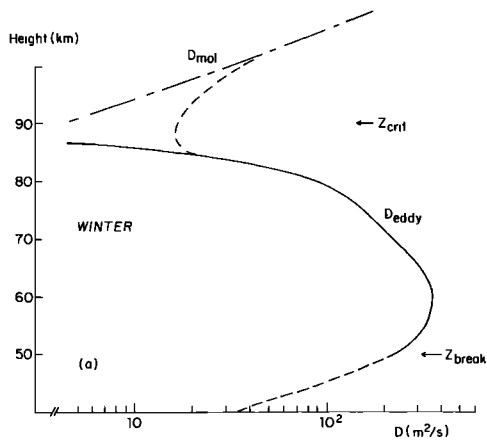


Fig. 5a

Fig. 5. Solid line shows predicted eddy diffusion coefficient based on breaking gravity waves. Simple dashed lines show schematic extrapolations as discussed in the text. The line consisting of alternating short and long dashes shows the molecular diffusion coefficient. (a) results for winter, (b) results for summer.

from which we obtain

$$\left(1 + \frac{P}{k^2}\right)^{1/2} \sim 1.54$$

or

$$\frac{l}{k} \sim 1.17 \tag{26}$$

While (26) ought not to be taken too seriously, the fact that l and k are about equal is not implausible. As a practical matter, it should be noted that a modest increase in l/k can significantly decrease both D_{eddy} and acceleration.

The guidelines for determining k , itself, are comparably uncertain. There does exist a lower bound of sorts on k . Namely, in the presence of rotation, vertical propagation occurs only when the doppler shifted frequency $|k(\bar{u} - c)|$ exceeds the Coriolis frequency, $f = 2\Omega \sin \theta$ (where Ω = earth's rotation rate and θ = latitude). Now, through the bulk of the mesosphere (near 40° latitude) $|\bar{u} - c| > 25$ m/s, and propagation will be possible for

$$k \geq \frac{2\Omega \sin 40^\circ}{25 \text{ m/s}} \approx 3.74 \times 10^{-6} \text{ m}^{-1} \tag{27}$$

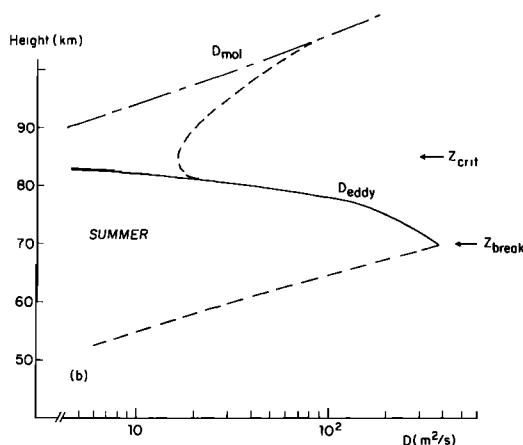


Fig. 5b

(40° is the approximate latitude for Figures 1 and 3). Finally, both surface topography and meteorological disturbances (both of which we have assumed are the basic sources of mesospheric gravity waves) tend to have red noise spectra. We will, therefore, take the lower bound for k given by (27) to also be the characteristic value of k . This value of k corresponds to a horizontal wavelength of 1680 km.

c. Eddy Diffusion

We evaluate (17) above z_{break} (appropriate to gravity waves) for the profiles of \bar{u} and \bar{T} shown in Figures 3 and 4. The results are shown in Figure 5. We have schematically continued D_{eddy} below z_{break} to indicate that nonbreaking gravity waves can still generate turbulence. We also show schematically an increase of D_{eddy} above the neighborhood of z_{crit} . The idea here is simply that while gravity waves may have a characteristic phase speed, c , they are not restricted to that phase speed. Weaker contributions at other phase speeds may break above that z_{break} appropriate to the bulk of the gravity wave energy. It is perhaps worth noting that the weaker gravity waves would break at a lower height if the bulk of the gravity wave spectrum were absent, since the weaker waves would not then be subject to damping by the turbulence generated by the stronger waves. Finally, one must recall that molecular diffusion increases as $1/\rho_0$ and reaches 0 (2×10^2 m²/s) by about 110 km. Molecular processes, of course, tend to produce diffusive separation rather than turbulent mixing.

Turning to the explicitly calculated part of Figure 5, we see that for both summer and winter, D_{eddy} peaks near z_{break} . Here z_{break} , as determined from Figure 1, occurs near 50 km in winter and 70 km in summer. In contrast to most models for eddy diffusion, Figure 5 shows a decrease in D_{eddy} with height above the neighborhood of z_{break} . The more precipitous decrease in summer is simply associated with the more rapid approach of \bar{u} to c in summer in the profiles shown in Figure 3. Whether such profiles are, in fact, applicable at any given time is open to question. In particular, during sudden warmings when an easterly flow regime sets on in the stratosphere, waves with $c = 0$ are effectively prevented from reaching the mesosphere and profiles for D_{eddy} should approach distributions similar to what is obtained for summer. This is what appears to be needed to produce enhanced D region ionization [Koshelev, 1976].

Interestingly, Allen et al. [1981] in a recent observation study of upper atmosphere composition, inferred a distribution of D_{eddy} very similar to the summer distribution in Figure 5. Their maximum value of D_{eddy} was about half of what is shown in Figure 5, but an increase in $(1 + P/k^2)^{1/2}$ of only 25% would eliminate this discrepancy.

Finally, we estimate D_{eddy} for the main propagating diurnal tidal mode. As already mentioned, the tide is expected to break at about 85 km. Since $c \approx -465$ m/s, \bar{u} is too small to be of much significance. Thus (17) reduces to

$$D_{\text{eddy}} \approx \frac{kc^4}{N^2(1 + P/k^2)^{3/2}} \frac{1}{2H} \tag{28}$$

According to CIRA (1972), $\bar{T} \approx 200^\circ\text{K}$ at 85 km and $d\bar{T}/dz \approx 0$. With these values (28) yields

$$D_{\text{eddy}} \approx 1.83 \times 10^2 \text{ m}^2/\text{s}$$

The diminution of D_{eddy} with height due to the variation of \bar{u} was not expected in this case. However, molecular viscosity

and conductivity rise rapidly in this region, and their role in damping the tide is similar to the role of D_{eddy} . Thus we must let D_{eddy} decrease with height so that the sum of molecular and eddy diffusion is given by the above value. This is shown in Figure 6. It should be repeated that although the roles of eddy and molecular diffusion in damping waves are essentially similar, the same cannot be said for their roles in transporting chemical constituents.

d. Wave-Induced Acceleration

As noted in section 2 and shown in Figure 1, $z_{\text{break}} \approx 50$ km in winter and $z_{\text{break}} \approx 70$ km in summer. Using the data in Figures 3 and 4 to evaluate (24) we obtain

$$-\frac{1}{\rho_0} \frac{d}{dz} \rho_0 u w \approx 135 \text{ m/s/day}$$

for $z_{\text{break}} < z < z_{\text{crit}}$ in summer and

$$-\frac{1}{\rho_0} \frac{d}{dz} \rho_0 u w \approx -102 \text{ m/s/day} \quad (29)$$

for $z_{\text{break}} < z < z_{\text{crit}}$ in winter.

The values given by (29) seem large, and there is ample room for uncertainty. In particular, it was suggested in section 3c that it might be advisable to increase $(1 + P/k^2)^{1/2}$ 25% to reduce our estimates of D_{eddy} by a factor of 2. Such a change would also reduce the values in (29) by a factor of 2. That acceleration is constant in the layer $z_{\text{break}} < z < z_{\text{crit}}$ can, of course, only be approximately true. Total wave absorption in the neighborhood of the critical level, for example, implies the existence of a small neighborhood of the critical level where accelerations exceed the values given by (24).

Application of (24) to the diurnal tidal mode (using the same values as in section 3c) yields

$$-\frac{1}{\rho_0} \frac{d}{dz} \rho_0 u w \approx -16.3 \text{ m/s/day} \quad (30)$$

for $z_{\text{break}} < z \approx 110$ km. Recall $z_{\text{break}} \approx 85$ km while 110 km corresponds to a region within 30° latitude of the equator. However, as was noted by *Andrews and McIntyre* [1976], the detailed latitudinal distribution of the momentum deposition can be more complicated.

It was earlier remarked that the accelerations in (29) seemed 'large.' Clearly, such accelerations are not, as a rule, directly observed in the \bar{u} field. However, it is not immediately clear whether the values in (29) are large compared to other contributors to the momentum budget. We might, for example, inquire what values of meridional velocity, \bar{v} , would allow Coriolis forces to balance the wave induced accelerations; that is,

$$2\Omega \sin \phi \bar{v} \sim 100 \text{ m/s/day}$$

or

$$\begin{aligned} \bar{v} &\sim 0 \left(\frac{100 \text{ m/s/day}}{2\Omega \sin \phi} \right) \\ &\sim 0 \left(\frac{8 \text{ m/s}}{\sin \phi} \right) \end{aligned} \quad (31)$$

Now according to CIRA (1972), such meridional winds are observed in the upper mesosphere (though one might question whether the observations really represent a zonal average). If

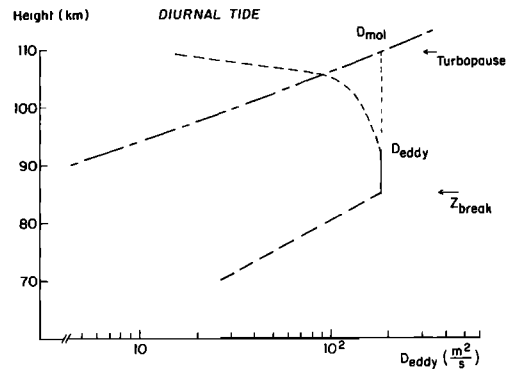


Fig. 6. Same as Figure 5, but for turbulence produced by the breaking of the diurnal tide.

this is so, then accelerations on the order of those given by (29) would be necessary to balance the 'observed' Coriolis torques.

Incidentally, the relatively small accelerations produced in the tropics could be balanced by the diffusion of momentum by the tidally induced eddy diffusion; this is not the case for the gravity wave-induced turbulence and acceleration in middle and high latitudes.

4. CONCLUDING REMARKS

For some years it has been accepted that tides and gravity waves propagating into the upper mesosphere from below are the major source of turbulence in the upper mesosphere. The purpose of the present paper has been to examine the implications of such a situation in some detail. The main propagating diurnal mode seems to be the primary contributor at tropical latitudes. Because of the high phase speed of this mode (~ 465 m/s), it is only slightly affected by the mean zonal flow of the atmosphere [*Lindzen*, 1971]. Wavebreaking appears to occur around 85 km, leading to a layer of enhanced eddy diffusion and wave induced acceleration ($0(-16 \text{ m/s/day})$) extending between 85 km and about 108 km above which height molecular transport dominates.

The situation at middle and high latitudes is more complicated. Gravity waves appear dominant at these latitudes, and if such waves originate in the troposphere (as seems likely) then their phase speeds will typically range from zero (appropriate to mountain waves, etc.) to typical tropospheric flow speeds. For such low phase speeds the effect of the mean zonal flow distribution will be crucial. The flow distribution will effectively determine which gravity waves (depending on phase speed) can reach the mesosphere and relatedly the amplitudes of mesospheric gravity waves and their breaking level. This effect, for example, implies that gravity wavebreaking will occur at greater altitude in summer than in winter (70 km versus 50 km). In addition, the turbulent diffusion and wave-induced accelerations no longer extend up to some level where molecular processes dominate. Instead, they are restricted to layers extending from the breaking level to some approximate critical level (where the mean flow, \bar{u} , equals or almost equals the wave phase speed). The latter are likely to be developed by the wave-induced accelerations themselves. Between the critical levels and the thermosphere we expect a sharp minimum in eddy diffusion.

The wave-induced accelerations provide an explicit source for the 'friction' needed to reverse mesospheric shears and,

relatedly, to reverse the pole-to-pole temperature gradient at the mesopause. Models without such friction develop neither of these features [Mahlman and Sinclair, 1979]. The wave-induced acceleration acts to bring the mean flow toward the wave phase speed. In this respect the acceleration is similar to Rayleigh friction, provided the waves have zero phase speed. In contrast to Rayleigh friction the acceleration is not proportional to $(\bar{u} - c)$; indeed, it can increase as $(\bar{u} - c) \rightarrow 0$ (i.e., as a critical level is approached).

There are, in the present study, any number of shortcomings. For example, the existence of D_{eddy} below z_{break} requires that there be some momentum deposition below z_{break} in contrast to our restricting the deposition to $z \geq z_{\text{break}}$. Similarly, in a calculation where $A(\theta)$ is specified instead of z_{break} (viz. (10)), this will alter the value of z_{break} . It is not difficult to include such effects (at least approximately), but their inclusion seems unwarranted in the present paper, which is meant to be primarily illustrative. More problematic is the fact that our mathematical development considered only the vertical propagation of gravity waves (vis. (1)). A realistic application of the present concepts may require considering the propagation of gravity waves in a two-dimensional inhomogeneous medium. This, however, remains to be investigated. It is hoped that, for many purposes, a reasonable specification of $A(\phi)$, and the use of our present formulas, will suffice.

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