

Mathematical Requirements for  
Atmospheric Dynamics  
AOSS 401  
Draft 1

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# Mathematical Tools

- Algebra and Trigonometry
  - Dependent and Independent variables
  - Algebraic manipulation of equations and inequalities
    - How to calculate units
  - Quadratic formula
  - Law of exponents
  - Logarithm and exponentials
  - Law of sines
  - Trigonometric identities
  - Equation for circle, parabola, hyperbola
- Series
  - Taylor series
    - Basic rules of linear approximation
    - Binomial series
    - Series to estimate sine, cosine, etc.

# Mathematical Tools

- Basics of vectors
  - Ability to determine components in orthogonal coordinate systems
  - Scalar (dot) product
  - Vector (cross) product
  - Div, Grad, Curl, del, del<sup>2</sup>
- Basics of calculus
  - Limits of small differences
  - Calculation of derivatives and partial derivatives
    - Derivatives of products and sums
    - Chain rule
    - Derivatives of logarithm, exponential, sine, cosine, *etc.*
  - Rules of integration
    - Definite versus indefinite integrals
    - Derivatives of logarithm, exponential, sine, cosine, *etc.*

# Where I found challenges

- Components of vectors
- Coordinate transformation
  - We did not formally look at curvilinear coordinates. This would be needed for the next course in the series.
- Solving these equations

$$\int_a^b \frac{dx}{x} = \int_a^b d \ln x \quad \frac{dT}{dt} = H - \lambda T$$

Would be useful.

(Required for graduate students)

- Contour integrals
- Given a differential equation, assume wave-like solutions, calculate the relationship between the parameters that are wave-like. (Dispersion relation)
- Stokes theorem, Gauss' Theorem ... integral theorems ...
- Complex exponentials and introduction to Fourier series for next sections in the book.

# Mathematical Expressions (for arbitrary $x$ )

$$f(x + \Delta x) = f(x) + \Delta x \frac{\partial f}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 f}{\partial x^2} + \dots$$

$$(1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$d \ln x = \frac{1}{x} dx, \ln x - \ln y = \ln \frac{x}{y}$$